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AMP ’18 Talk

Learning a frequency-matching grammar together with lexical idiosyncrasy: MaxEnt versus mixed-effects logistic regression

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SPEAKERS KNOW AGGREGATE GENERALIZATIONS *AND* IDIOSYNCRASIES

1. Language learners *frequency match* to statistical generalizations across the lexicon[1]

- E.g., Hungarian vowel harmony (Hayes & Londe 2006): dative forms takes *-nɛk* or *-nɔk*, depending on backness of preceding stem vowel. Stems ending in...
  - front V tend to take *-nɛk*: [kɛ̱rt-nɛ̱k] ‘garden’-DAT, [y̱ʃt-nɛ̱k] ‘cauldron’-DAT
  - back V tend to take *-nɔk*: [ɔblɔ̱k-nɔ̱k] ‘window’-DAT, [bi:ro̱:-nɔ̱k], ‘judge’-DAT

- Corpus study of monosyl. stems ending in front, unrounded V: **92%** take *-nɛk*; **8%** *-nɔk*.

- In wug tests, speakers presented with fake monosyllabic stems with a front unrounded vowel, in aggregate, ***closely frequency-matched to the 8% -nɔk rate***.

(1)

	-nɛk	*-nɔk*
Corpus rate:	92%	8%
Wug test rate:	93%	7%

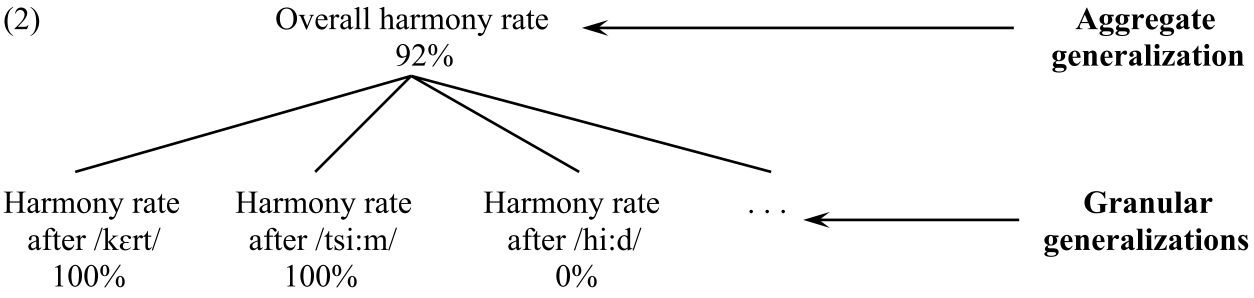
(Hayes & Londe 2006)

2. But language learners also know lexical idiosyncrasies

- Speakers know *which* attested words harmonize, versus not (Hayes & Londe 2006).
- French speakers even track *morpheme-specific rates* of liaison (Zymet 2018).

3. Language learners thus internalize *nested hierarchy of generalizations*:

(2)



Overall harmony rate
92%

Aggregate generalization

Harmony rate after /kɛrt/ 100%

Harmony rate after /tsi:m/ 100%

Harmony rate after /hi:d/ 0%

. . .

Granular generalizations

- Recent MaxEnt model (Moore-Cantwell & Pater 2016, Zuraw & Hayes 2017, Tanaka 2017):
  - General constraint to frequency match general trend across the lexicon (HARMONIZE)
  - Lexical constraints for specific attested words (HARMN(kɛrt), DON’T-HARMN(hi:d))

[1] Frisch, Broe, & Pierrehumbert 1996; Coleman & Pierrehumbert 1997; Eddington 1998, 2004; Berkley 2000; Zuraw 2000, 2010; Bailey & Hahn 2001; Frisch & Zawaydeh 2001; Albright 2002; Albright & Hayes 2003; Ernestus & Baayen 2003; Hayes & Londe 2006; Becker 2009; Hayes, Zuraw et al. 2009; *et seq*.

4. Today: Modeling learning of frequency-matching grammar with lexical idiosyncrasy

- Learning simulations reveal that **lexical constraints are too powerful in MaxEnt**:
  - o A priori, general constraint and set of lexical constraints considered *equally viable* hypotheses about the data in MaxEnt;
  - o at high levels of learning, lexical constraints come to explain every form in dataset, rendering the general constraint superfluous and ineffective.
  - o General constraint weight plummets to zero, failing to predict learners’ frequency-matching abilities in wug tests.

- **Solution**: Switch from MaxEnt—essentially single-level logistic regression model—to hierarchical **MIXED-EFFECTS LOGISTIC REGRESSION MODEL**.
  - o General/lexical constraints no longer equal: general constraints preserved as fixed effects; lexical constraints form random effect.
  - o Hierarchical model captures *hierarchy of generalizations*: aggregate trend + idiosyncrasies of individual words.
  - o We apply mixed model to variable Slovenian palatalization—with promising results.

MAXENT: THE GRAMMAR-LEXICON BALANCING PROBLEM

5. MaxEnt (Smolensky 1986, Goldwater & Johnson 2003, Hayes & Wilson 2008, *et seq*)

- Constraints have numerical weights instead of rankings;
- surface forms assigned probabilities as function of weights.
- Learning rooted in *accuracy* and *simplicity*: model takes constraints, finds best weights it can to fit overall rates in dataset; useless constraints discarded—weight set to zero.

❖ But MaxEnt fits to *overall rates*; investigators hadn’t tried to get MaxEnt to also learn which words are un/exceptional until recently. The new approach:
  - o General constraints for overall trend, lexical constraints for specific-word behavior
  - o Moore-Cantwell & Pater (2016), Zuraw & Hayes (2017), Tanaka (2017), *inter alia.*

6. Does the MaxEnt approach to learning frequency matching & idiosyncrasy work?

- Suppose we have 46 regulars, 4 irregulars—irregularity rate of **8%**.
- 3 constraints: BEREG, BELEX(regulars), BELEX(irregulars) initiated at 0 weight
- If we want to learn the dataset better? Multiply frequencies by 10. Worse? By 0.1.
- (Caveat: introduced *a little* variability: 0.001% /regs/ surface [irreg]; 0.001% /irregs/ as [reg])

UR	SR	Freq.	BEREG 0	BELEX(reg) 0	BELEX(irreg) 0
/Regular/	Regular:	≈ 46			
	Irregular:	≈ 0	-1	-1	
/Irregular/	Regular:	≈ 0			-1
	Irregular:	≈ 4	-1		

Table 1: *MaxEnt input*

- We want MaxEnt to learn weights such that:
  - in wug test, irregular form picked **~8%** of time;
  - *attested* words (=words in learner input) are pronounced correctly ~100% of time.
  - *w*(BEREG) = 4, *w*(BELEX-reg) = 3, *w*(BELEX-irreg) = 11 gives great results.

- But does MaxEnt *learn* good weights from the input? Let's run learning simulation using Excel Solver, which can fit parameters of nonlinear models (Fylstra et al. 1998, Harris 1998):
  - Trial run by using data in Table 1,
  - and multiplying frequencies of the dataset by a small factor (0, 0.001, etc.)—we call this "childhood". We learn poor weights (*w*(BELEX-irreg)=0.5) that don't fit the data.
  - After each trial, increase frequency factor slightly, get new weights—"adolescence".
  - When frequencies get large and we think we have final weights—"adulthood".

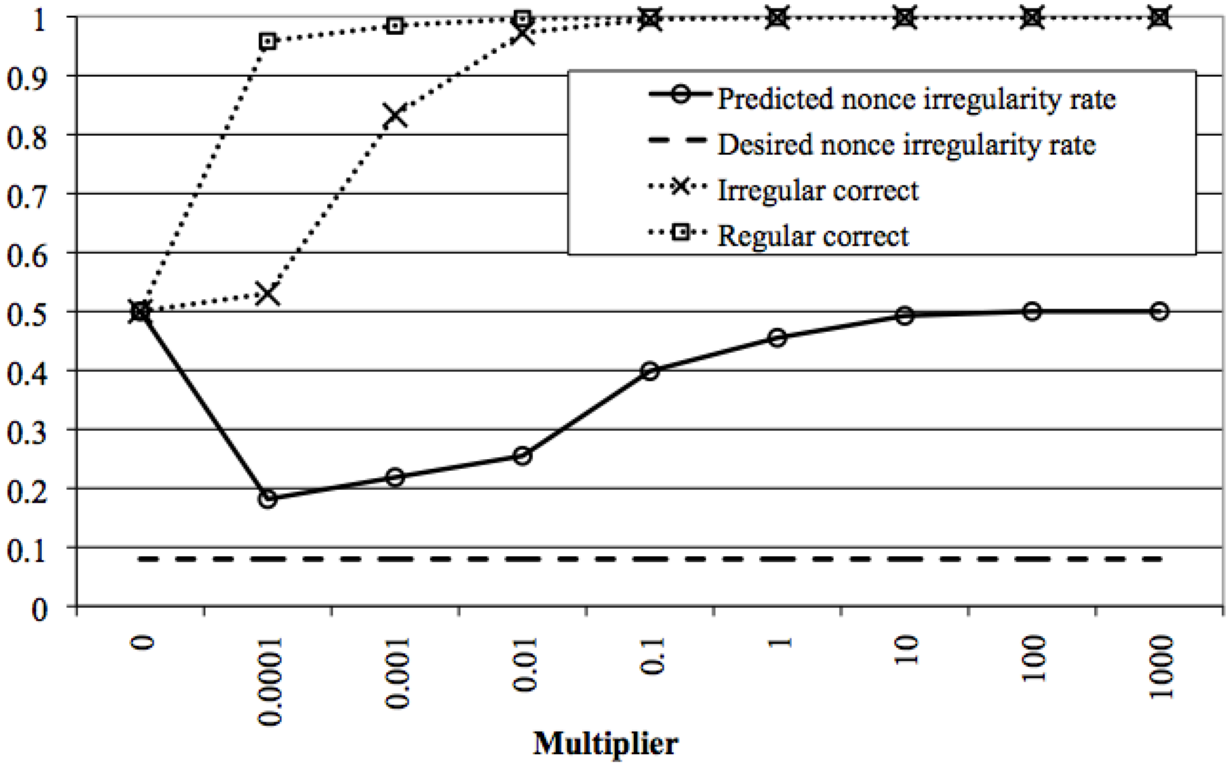


Figure 1: *MaxEnt fails to learn generalization together with idiosyncrasy* (σ = 100)

- With frequency factor 0, baby learns 0-valued weights, prefers 50/50 regular/irregular.
- As child grows (freq. factor 0.0001), rapidly starts to learn regulars, slowly tackling irregulars.
- Early in learning, BEREG is used to explain much of the variation—we see that with low nonce irregularity rate.
- But eventually BELEX constraints grow very high, coming to explain entire set of attested data. BEREG comes to explain increasingly *less* of data, eventually *perishing*.
- By adulthood (freq. mult. 1000), **BEREG sinks to 0, rendered superfluous/ineffective.**
- At that point, the learner selects regulars/irregulars at 50/50 rate in wug tests—forgetting the grammar entirely. See Appendix for simulation output numbers.

7. Hence, the GRAMMAR-LEXICON BALANCING PROBLEM. In MaxEnt...

- A priori, general constraint/set of lexical constraints *equally viable hypotheses* about data,
- **consequently lexical constraints too powerful**: lexical constraints learn each word's behavior before general constraint matches overall 8% trend, at which point frequency matching ceases and the general constraint becomes ineffective.
- No phonological learning; just lexical learning. Implausible that speakers fail wug tests once they learn lexicon (see Shademan 2007 for learning in elderly).
- We need a theory that, while accounting for idiosyncrasies, ***preserves grammar***.

- We search for model possessing **GENERALITY BIAS**: general, grammatical constraints must be privileged to lexical constraints in the learning process.
  - Adjusting MaxEnt penalty term does not work: dividing σ's by 10 = dividing freq. multiplier by 100—merely *delays* overfitting (see Appendix).
  - High σ(BEREG)/low σ(BELEX) so far does not work; overfits at higher multiplier.

LEARNING LEXICAL VARIATION WITH MIXED-EFFECTS LOGISTIC REGRESSION

8. What about the hierarchical MIXED-EFFECTS LOGISTIC REGRESSION model?

- Similar to binomial logistic regression, except constraints hierarchically arranged as follows:
  - **Fixed effects**: those constraints that we are actually interested in—e.g., phonological constraints, yielding the statistical generalizations in the dataset
  - **Random effects**: constraints that capture the idiosyncrasies in the data—deviations from generalizations captured by fixed effects.
  - We might call this **Mixed Effects Maximum Entropy Harmonic Grammar**.
- Used widely in science to capture trends & idiosyncrasies in variable datasets;
- Linguists employ random intercepts to measure by-word/lexical class idiosyncrasy (Fruehwald 2012, Zuraw & Hayes 2017, Smith & Moore-Cantwell 2017, *inter alia*);
- Shih & Inkelas (2016)/Shih (2018) even adopt multilevel model as theory of learner.

9. Mixed models *hierarchical*: random effects "depreciated" relative to fixed effects

We have a fixed effect—a general constraint—BEREGULAR, whose weight is estimated based on average harmony rate across the entire dataset—**92%**.

(3a) *w*(BEREG): $\mu_{all\ words}$

- We want this weight to accurately estimate the average rate across all words, as that would be a **frequency-matching grammar**, mimicking human behavior in wug tests.

We have a random effect (random intercept) consisting of weights for lexical constraints:

- *w*BELEX–irreg1, for example, estimated by rate irregular1 (0.001) ...
- ***and* by overall rate across dataset**:

(3b) *w*(BeLex–irreg1): $\lambda_{irregular1} * \mu_{irregular1} + \left(1 - \lambda_{irregular1}\right) * \mu_{all\ words}$

Raudenbush & Bryk (2012), Snijders & Bosker (2012)

- λ: value between 0 and 1, depends on size of the group: *w*(BeLex–irreg1) will be determined more by $\mu_{irregular1}$ if data have lots of irreg1 tokens rather than few.
    - Predicts more idiosyncrasy with frequent forms, but more grammatical behavior with infrequent forms (Morgan & Levy 2016, Moore-Cantwell & Smith 2016).
- **Think of mixed models as follows**: fixed effect weights predicts overall rate, and random effect weights predict ***word-specific offsets*** from overall rate.
- Source of the generality bias: lexical constraint weights *depend* on overall average rate.

10. Mixed model performs well on strict exceptionality dataset

We want the learning model to predict:

- With BeReg, the average rate across all Words in the dataset—hence a frequency-matching grammar
- With BeLex–reg/BeLex–irreg, the specific rates for every word.

We run a model of the dataset using the `glmer` function of the *lme4* package R.

- weight of BeReg is the general intercept
- weight of BeLex constraints are the coefficients of the levels of the random intercept.

To extract predicted nonce rate from model, you cannot simply plug *w*BeReg into inverse logit—rather, you must "average" over the levels of the random intercept (Pavlou et al. 2015).

- This involves a complex integral that cannot be calculated analytically;
- Zeger et al. (1998) provide a good approximation:
    - *c* is constant equal to $\frac{16\sqrt{3}}{15\pi}$
    - τ^2 is variance of random intercept (14.77)

(4)

$$\frac{\exp\left(\frac{w\text{BEREG}}{\sqrt{c^2\tau^2+1}}\right)}{1+\exp\left(\frac{w\text{BEREG}}{\sqrt{c^2\tau^2+1}}\right)}$$

(5) **Results**:

Word	*w*BELEX	***Actual rate***	***Predicted rate***
reg1	0.69	**0.999**	**0.999**
reg2	0.69	**0.999**	**0.999**
...			
reg46	0.69	**0.999**	**0.999**
irreg47	-12.46	**0.001**	**0.002**
...			
irreg50	-12.46	**0.001**	**0.002**

***w*BEREG = 6.167**

OVERALL IRREGULARITY RATE: 8%
PREDICTED NONCE IRREGULARITY RATE: 7.4%

This model:

- Predicts word-specific rates—learns lexical effects.
- Frequency-matches overall rate—mimicking subjects in wug tests—without lexical constraints *starving* general constraint. ***Grammar sustained after lexical learning.***

11. Mixed model performs well on dataset with different lexical rates

- Consider the following: twelve words, each with 1000 tokens, with the different tokens undergoing, say, harmony, at different rates.

Word	Rate	Word	Rate	Word	Rate
1	0.00	5	0.30	9	1.00
2	0.00	6	0.80	10	1.00
3	0.10	7	0.90	11	1.00
4	0.20	8	1.00	12	1.00

Average over all rates: **0.61**

Table 2: *propensity dataset*

- Two kinds of constraints:
  - APPLY (HARMONIZE), whose weight should frequency match to **61%** overall rate
  - APPLY-Word1, ..., APPLY-Word12, assists with specific rates

(6) **Results:**

Word	*wLex constr.*	***Actual rate***	***Predicted rate***
Word1	-16.56	**0.000**	**0.000**
Word2	-16.56	**0.000**	**0.000**
Word3	-7.32	**0.100**	**0.100**
Word4	-6.51	**0.200**	**0.200**
Word5	-5.97	**0.300**	**0.300**
Word6	-3.74	**0.800**	**0.800**
Word7	-2.93	**0.900**	**0.900**
Word8	7.14	**1.000**	**0.999**
Word9	7.14	**1.000**	**0.999**
Word10	7.14	**1.000**	**0.999**
Word11	7.14	**1.000**	**0.999**
Word12	7.14	**1.000**	**0.999**

***w*HARMONIZE = 5.130**

OVERALL AVERAGE APPLICATION RATE: 0.61
PREDICTED APPLICATION RATE TO NONCE WORDS: 0.66

- I tried MaxEnt on this dataset:
  - Outcomes similar to other dataset, except *w*APPLY vacillates/plummets to 0 at high levels of lexical learning—see Appendix.
  - See Zymet (2018) for further details.

APPLYING THE MIXED MODEL TO VARIABLE SLOVENIAN PALATALIZATION

- For example, only *some* suffixes trigger it.

(7a)

Stem		Triggering suffix /-it͡sa/		Non-triggering suffix /-inja/	
lu**k**-a	port-GEN	lu**tʃ**-it͡sa	port-DIM	lu**k**-inja	port-DIM
bo**g**-a	god-GEN	bo**ʒ**-it͡sa	god-DIM	bo**g**-inja	god-DIM

  - From Toporišič (1997/2000): Of 200 suffixes, only a handful trigger palatalization.

- Different palatalizing suffixes trigger at different rates, suggesting suffix identity plays role:

(7b)

/luk-itʃ/, port-DIM	/luk-ina/, port-ABS	/luk-it͡sa/, port-DIM
lu**tʃ**-itʃ, **18%** (558/3147)	lu**tʃ**-ina, **50%** (50/100)	lu**tʃ**-it͡sa, **98%** (39/40)
luk-itʃ, 82% (2589/3147)	luk-ina, 50% (50/100)	luk-it͡sa, 2% (1/40)

- *Stems* undergo at different rates before same suffix, suggesting stem identity plays role.

(7c)	Stem		Stem before diminutive -it͡sa		
	obla**k**-a	'cloud'-GEN	obla**tʃ**-it͡sa	'cloud'-DIM	*Undergoer*
	nɔ**g**-a	'leg'-GEN	nɔ**g**-it͡sa ~ nɔ**ʒ**-it͡sa	'leg'-DIM	*Vacillator*
	ja**k**-a	'yak'-GEN	ja**k**-it͡sa	'yak'-GEN	*Non-undergoer*

10. Jurgec (2016) on Slovenian palatalization

- Jurgec extracted words with velar-final stem + palatalizing suffix from two dictionaries:
  - *Dictionary of Standard Slovenian* (Bajec 2000; 110,000 word types)
  - *Slovenian Orthographic Dictionary* (Toporišič 2001; 130,000 word types).

- To obtain token rates for each word, he fed them into *Gigafida* (Logar-Berginc et al. 2012):
  - Text corpus w/ ~1.2 billion tokens from written sources ca. 1990–2011.
  - His resulting data set included ~5.7 million tokens.

- Jurgec suggests phonological factors condition variation in his data:
  - Suffixes with front vocoids trigger more regularly
  - Velars *k*, *g* undergo more regularly than *x*.
  - Suffixes with *t͡s* trigger less regularly.
  - Palatalization regularly applies to avoid geminate in /...{k, g}+k/ (-*k* = -DIM)
  - Palatalization blocked by distant postalveolars earlier in the stem.

- Jurgec gives MaxEnt account of *phonological* conditioning; suffix idiosyncrasy encoded with **[+/- Pal'n]**—only picks out suffixes with *any degree* of palatalization.
  - But he *does* observe suffix-specific rates in his study—lexical propensities left to further research.

12. Building upon Jurgec (2016): a corpus investigation into lexical propensities

- I show that:
  - Morphemes have **LEXICAL PROPENSITIES**: suffixes trigger at different rates, and stems undergo at different rates, patterning across an entire spectrum (**[0.7 Pal'n]**).
  - Mixed model encodes propensities while frequency matching to trends.

- Extraction method similar to Jurgec:
  - Words consisting of velar-final stems + palatalizing suffix extracted from *Dictionary of Standard Slovenian*.
  - Each extracted stem concatenated with each of nine suffixes, creating hypoth. words
  - Fed each word into *Gigafida*, extracting frequencies/token rates
  - Yielded ~3 million tokens of words either undergoing/not undergoing palatalization

- I calculated palatalization rates for each suffix. /ag/ undergoes 22% of time before -/je/, /kak/ 99% of time; average rate before -/je/ is 88%.

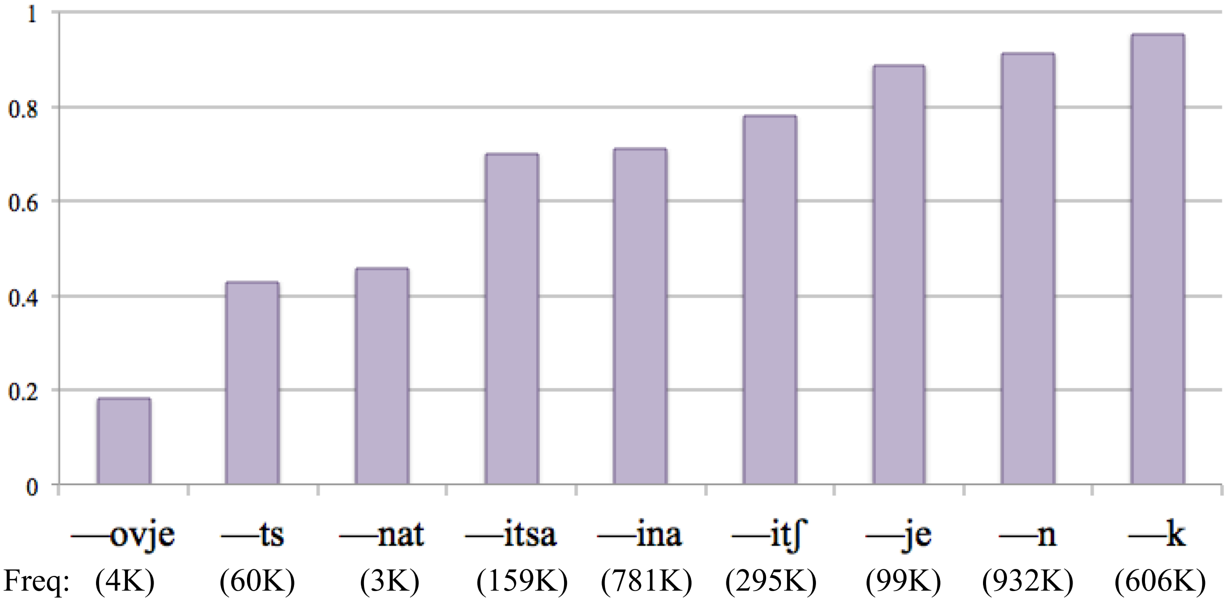


Figure 2a: *palatalization rates for each suffix*

What about stems? A histogram of rates across 246 stems occurring before at least four suffixes:

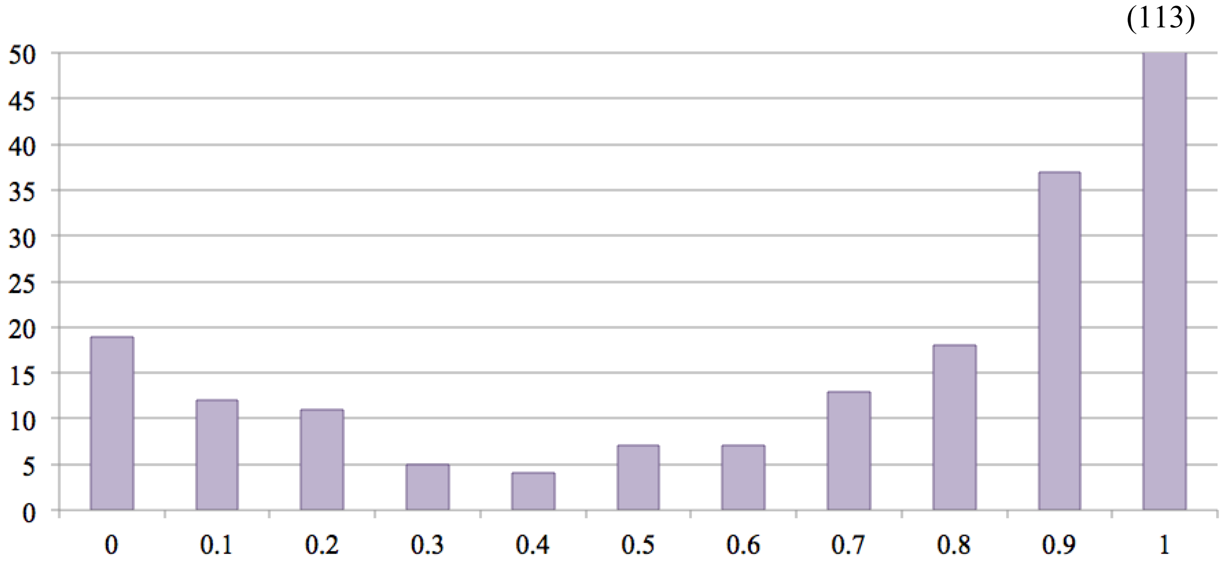


Figure 2b: *Histogram of stem palatalization rate frequencies*

- Results suggest morphemes have **LEXICAL PROPENSITIES**: suffixes trigger at different rates, and stems undergo at different rates, patterning across an entire spectrum.

- We use mixed-effects logistic regression to encode morphemes on a spectrum (**[0.7 Pal’n]**)—significantly improves model fit relative to binary scale (**[+/- Pal’n]**).
  - Models run using `glmer` functions of *lme4* package (Bates & Maechler 2011) in R.

- In this handout, we focus on/compare performance of following logistic models:
  - **Baseline Model**, containing fixed effects for:
    - Stem-final velar identity (*k, g, x*)
    - Whether suffix begins with a front vocoid
    - Whether stem contains an earlier post-alveolar
    - Whether the suffix contains a post-alveolar affricate
    - (Contains random effect for whole word; thus we’re regressing over frequency-weighted types.)
  - **Stem+Suffix Model**, containing:
    - all factors in Baseline Model
    - plus stem identity and suffix identity, ***encoded as random intercepts***.

- Models compared using Akaike Information Criteria (AIC; Akaike 1973), which scores models based on number of parameters and fit to the data: **lower score = better**.
  - See Bolker et al. (2009) for justification on using this to compare mixed models.

13. Results of Baseline Model

- Stem-final velar identity significant: *k* > *g* (seemingly a faith effect: k → tʃ but g → ʒ)
- Geminate avoidance significant
- ʃ...tʃ+ avoidance significant
- Suffix with *ʦ* significantly associated with lower rates
- `frontvocoid` not significant
- AIC: **8767.8**

```
             Estimate Std. Err.   z value       p
Intercept      3.95    0.47         8.29     <0.001 ***
ref: consg
consx          1.59    0.66         2.41      0.015 *
consk          1.96    0.47         4.11     <0.001 ***
kk             4.94    0.80         6.12     <0.001 ***
frontvocoid   -0.52    0.39        -1.32      0.183 (n.s.)
S...S         -1.67    0.78        -2.12      0.033 *
suff.with.ts  -3.53    0.46        -7.55     <0.001 ***
```

Output 1a: *Baseline Model results for Slovenian palatalization*

13. Results of Stem+Suffix Model

- Significant *k* > *g* effect and geminate effect, but no ʃ...tʃ+ effect or suffix-with-*ʦ* effect
- **Stem and suffix variances highly positive**—suggest stem and suffix condition variation
- Stem variance bigger than suffix variance: maybe undergoers louder than triggers; or linearly-first-element bias; or just relative morpheme counts. Feel free to ask in Q&A.
- AIC value: **7801.5** — substantial reduction from Baseline Model’s **8767.8**;

```
Random effects:
 Groups Name        Variance Std.Dev.
 stem   (Intercept) 68.06    8.25
 suffix (Intercept) 19.54    4.42
Number of obs: 2940918; words: 4822; stems: 2720; suffixes: 9

Fixed effects:
              Estimate Std. err. z value   p
Intercept:     1.15      2.24       0.51   0.60
ref: consg
-------------------------------------------------------
consx          2.36      1.00       2.35   0.02     *
consk          2.59      0.69       3.75  <0.001    ***
-------------------------------------------------------
k+k            7.94      1.32       6.01  <0.001    ***
-------------------------------------------------------
frontvocoid    2.72      3.01       0.90   0.366
S...S         -1.20      1.12      -1.06   0.284
suff.with.ts  -1.88      3.58      -0.52   0.598
```

Output 1b: *Stem+Suffix Model results for Slovenian palatalization*

14. AICs suggest suffix *and* stem identities matter ($p < 0.001$ by likelihood ratio test)

- Baseline Model AIC: **8767.8**
- Suffix-Only Model AIC: **8283.7**
- Stem-Only Model AIC: **8128.9**
- Stem+Suffix Model AIC: **7801.5**
- See Zymet (2018) for further elaboration on all these models.

15. Mixed model learns *the phonology*, frequency matching to statistical trends

- Matching to overall palatalization rate for *k*-final stems, and *g*-final stems
- Predicts *k* > *g* effect
- Predicts geminate-avoiding palatalization

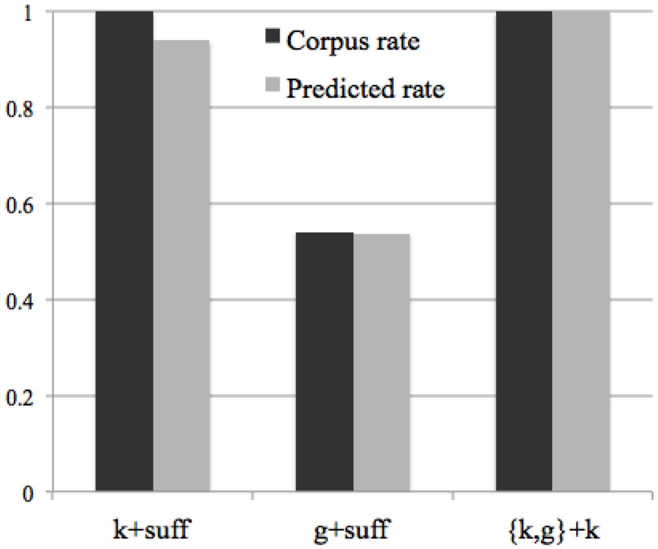


Figure 3a: *model succeeds in predicting phonological trends*

16. The mixed model learns lexical propensities

- Fares well in predicting suffix-specific rates:

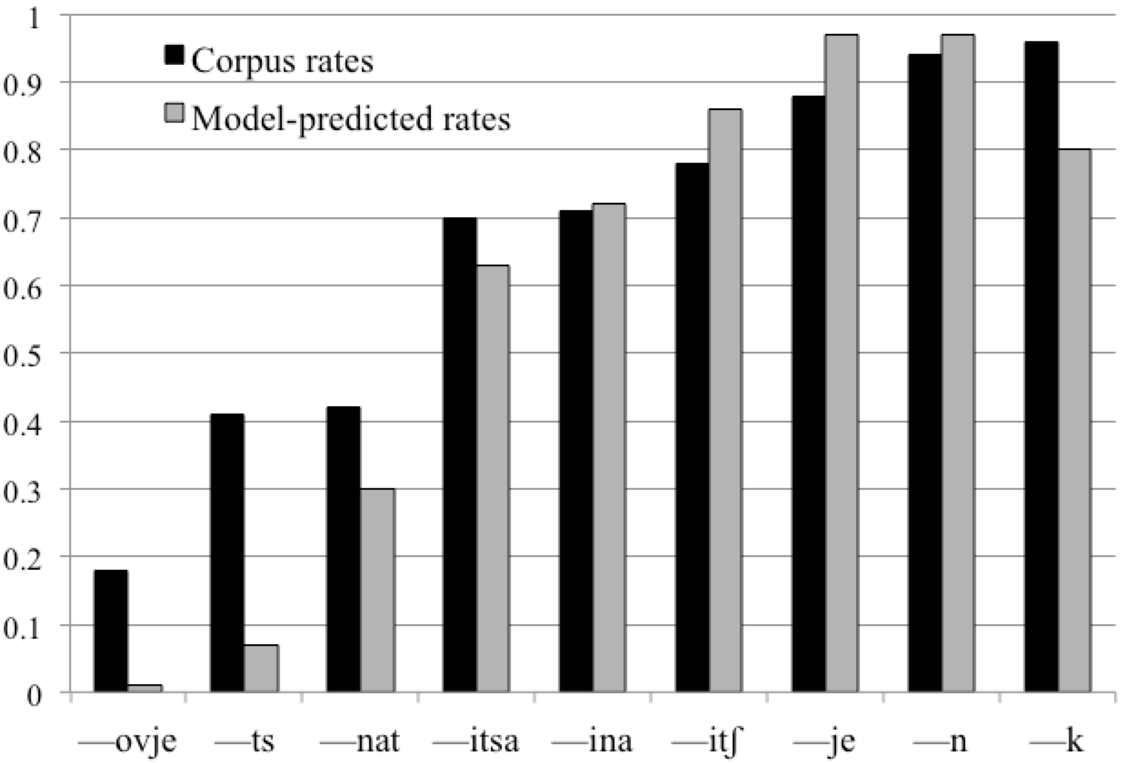


Figure 3b: *model-predicted suffix rates generally match corpus rates*

- I submit **mixed-effects logistic regression** as viable approach to modeling lexical variation—to learning of frequency-matching grammar with lexical propensities.

CONCLUSION

- Language learners internalize nested hierarchy of generalizations:
  - they can frequency match to aggregate statistical generalizations across the lexicon,
  - but also know which words are idiosyncratically exceptional, and which are not.

- MaxEnt/single-level regression doesn't recognize *hierarchicality of generalities*. I suspect problem is broader than just lexical variation:
  - If learner knows two groups of data have different rates,
  - and averages over rates when encountering novel data lying outside both groups,
  - then how could we model this averaging if we have accurate model of group rates?
  - MaxEnt: specific constraints enough to explain data, general constraint superfluous.

- Mixed-effects logit/mixed-effects MaxEnt surmounts balancing problem:
    - *Hierarchical* theory for a hierarchy of generalizations
    - Idiosyncratic effects of vocabulary subordinated to broad effects of grammar
    - Prior studies suggest it as potential model; today I give reason *why* this should be our theory of language competence.

- Future questions I hope to work on:
    - How should hierarchical theory look—e.g., how to plug random intercept into theory?
    - Exactly what constraints/kinds of constraints should be considered fixed vs. random?
    - How to expand mixed-effects MaxEnt to cover more than just the binomial case?
    - How to get >2 levels of generalization?

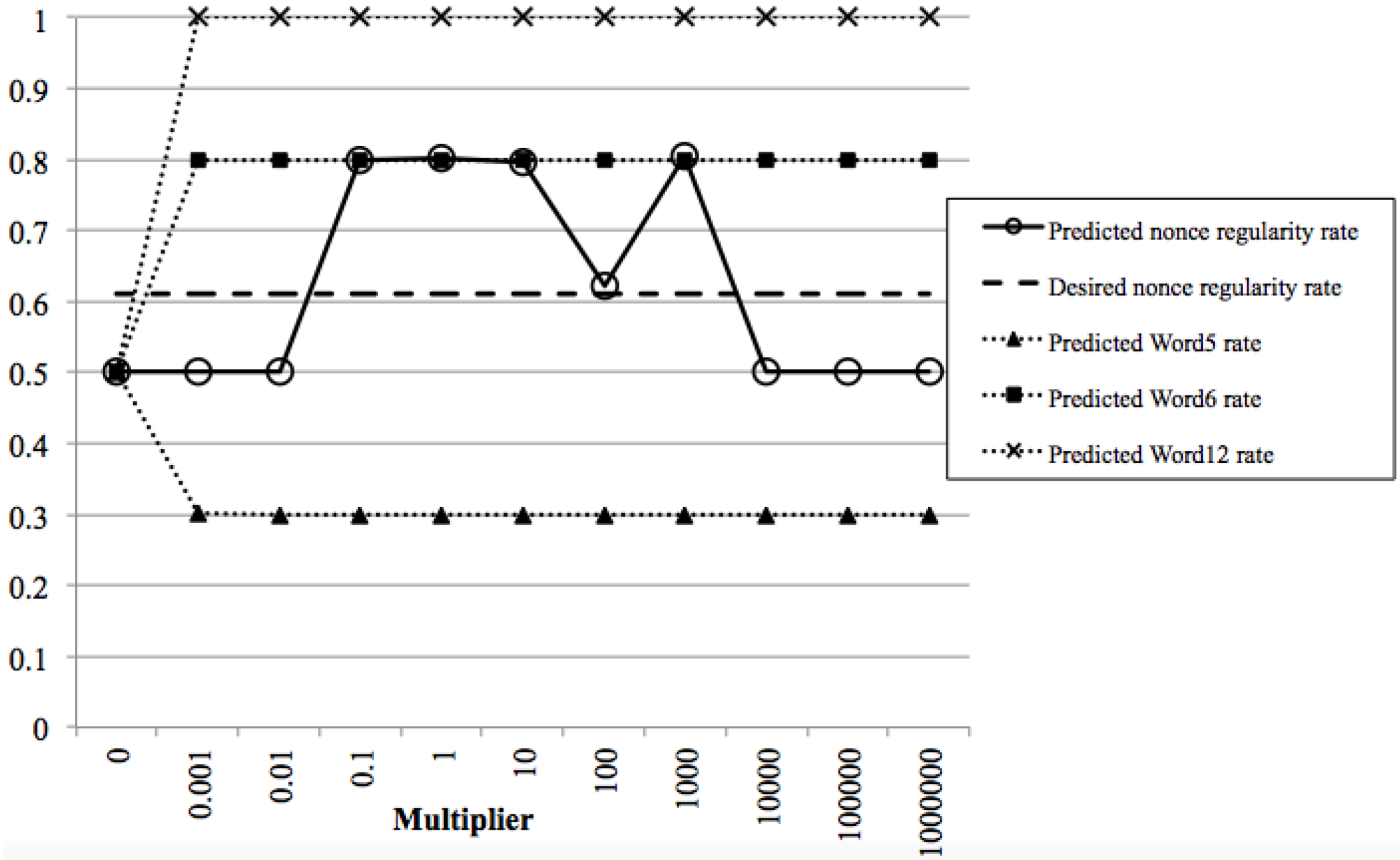
Appendix

FAILED LEARNING SIMULATION IN MAXENT (strict exceptionality, 8% irregularity rate)

Freq. multiplier	**Be Reg**	BeLex (regs)	BeLex (irregs)	Regular correct	Irreg. correct	**Nonce irreg. rate**
0	**0**	**0**	**0**	0.5000	0.5000	0.5000
0.0001	**1.50**	**1.62**	**1.62**	0.5304	0.9582	0.1815
0.001	**1.27**	**2.88**	**2.88**	0.8331	0.9845	0.2185
0.01	**1.07**	**4.63**	**4.63**	0.9723	0.9966	0.2548
0.1	**0.41**	**6.22**	**5.82**	0.9955	0.9986	0.3986
1	**0.17**	**6.69**	**6.78**	0.9986	0.9989	0.4551
10	**0.02**	**6.87**	**6.89**	0.9989	0.9989	0.4925
100	**0**	**6.90**	**6.90**	0.9989	0.9989	0.5000
1000	**0**	**6.90**	**6.90**	0.9989	0.9990	0.5000

Table A1: *MaxEnt learning simulation output numbers for strict exceptionality data*

FAILED LEARNING SIMULATION IN MAXENT (propensities dataset, 61% applic’n rate)



Freq. multiplier	APPLY	FAITH$_5$	APPLY$_6$	APPLY$_{12}$	**Pred. nonce rate**	Pred. Word5 rate	Pred. Word6 rate	Pred. Word12 rate
0	0.00	0.00	0.00	0.00	**0.50**	0.50	0.50	0.50
0.001	0.00	0.84	1.39	11.19	**0.50**	0.30	0.80	1.00
0.01	0.00	0.85	1.39	10.64	**0.50**	0.30	0.80	1.00
0.1	1.39	2.23	0.00	8.77	**0.80**	0.30	0.80	1.00
1	1.40	2.25	0.00	11.85	**0.80**	0.30	0.80	1.00
10	1.37	2.22	0.02	16.17	**0.80**	0.30	0.80	1.00
100	0.50	1.34	0.89	8.25	**0.62**	0.30	0.80	1.00
1000	1.41	2.27	0.00	6.16	**0.80**	0.30	0.80	1.00
10000	0.00	0.85	1.39	12.60	**0.50**	0.30	0.80	1.00
100000	0.00	0.85	1.39	11.84	**0.50**	0.30	0.80	1.00
1000000	0.00	0.85	1.39	11.84	**0.50**	0.30	0.80	1.00

Table A2: *MaxEnt learning simulation output numbers for propensity data*

OVERFITTING OUTCOME GENERAL ACROSS MAXENT PENALTY SETTINGS

- E.g., multiplying σ's by 10 yields same result as multiplying frequency multiplier by 100.
- Evident in the table below, which presents results of a series of learning simulations of the strict exceptionality dataset from above (but only fitting the weight of BeReg to it).
- Hence decreasing σ merely has the effect of delaying learner overfitting

	σ = 1		**σ = 10**		**σ = 100**	
	irreg. rate	weight	irreg. rate	weight	irreg. rate	weight
***m* = 0.01**	0.4748	0.1008	0.1127	2.0629	0.0213	3.8258
***m* = 1**	0.1127	2.0629	0.0213	3.8258		
***m* = 100**	0.0213	3.8258				

Table : *identical learning outcomes across different values of m and* σ

- Manipulating μ also has no effect—yields same learning outcome as if we set μ = 0.
- What about high σ(BeReg) and low σ(BeLex)?
- I tried it on a few strict exceptionality datasets (but not including the one given in this handout...), and so far the results are negative:
- Setting σ = 1,000 for BeRegular and σ = 10 for the lexical constraints, for example, still yielded overfitting, albeit at a high frequency multiplier.

COEFFICIENTS FOR STEMS AND SUFFIXES IN SLOVENIAN

Suffix	Rate
-ovje	-4.05
-ina	-1.27
-nat	-0.40
-itʃ	-0.38
-ts	-0.16
-itsa	0.16
-k	0.58
-je	1.48
-n	4.03

Stems (sample)	Rate
trak-	-5.34
tramik-	0.00
tradicionalistik-	0.55
tragikomik-	1.14
travmatik-	1.30
tragik-	2.31

- Coefficients run the gamut, suggesting gradience.
- Suffix coefficients generally track suffix rates we saw toward the beginning.

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