Computational Complexity and Sour-Grapes-Like Patterns

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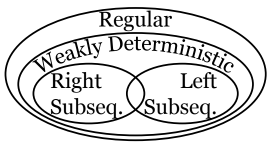
Introduction

- Input-output mappings can be classified hierarchically by computational complexity (Chomsky 1956)



- All attested phonological mappings are a proper subset of the class of regular input-output mappings (Heinz 2011)
- (Un)attestedness of certain phonological patterns can be attributed to computational complexity of input-output mappings (Heinz & Lai 2013, Jardine 2016)
- Sour-grapes-like spreading: spread phonological property throughout domain or not at all (Padgett 1995; Wilson 2003)

Proposals

Different sour-grapes-like patterns characterized by different degrees of computational complexity:

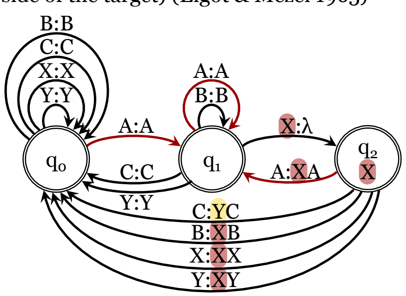
1) **False sour grapes** (attested) is relatively less complex due to zone of predictability local to potential triggers of spreading
2) **True sour grapes** (unattested) has no zone of predictability and is relatively more complex

Computational Complexity

- Input-output mapping of strings can be described by transformational rules or by finite state transducers
- Properties of rules/transducers indicate computational complexity of input-output mappings
- All regular mappings can be decomposed into left and right subsequential mappings (those with unbounded amount of material on only one side of the target) (Elgot & Mezei 1965)

Left subsequential:
X→Y/A(B)$_0$__C

Right subsequential:
X→Y/C__(B)$_0$A



- Weakly deterministic mappings (Heinz & Lai 2013) can be decomposed into left- and right-subsequential functions that:
  - Do not change the number of symbols in a string
  - Are alphabet-preserving (do not introduce new symbols)

Copperbelt Bemba Tone Spreading

- Copperbelt Bemba (Bantu; Zambia) exhibits ternary and unbounded spreading of H tones (Bickmore & Kula 2013)
- Final H spreads unboundedly to following tone bearing units:

H → H H H H
/bá-ka-fik-a/ ⟶ [bá-ká-fík-á] 'they will hate'

- Non-final H spreads only to two additional tone bearing units:

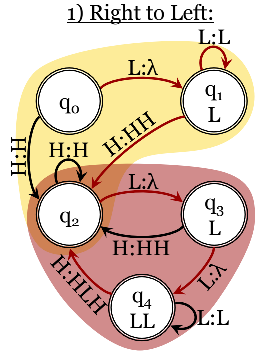
H H → H H H L L L H
/bá-ka-londolol-a kó/ ⟶ [bá-ká-lóndòlòl-à kó]
'they will introduce them'

- Previous claim: sour-grapes-like unbounded tone spreading in Copperbelt Bemba is not weakly deterministic (Jardine 2016)
- Whether H triggers unbounded spreading is not known until rest of word is scanned for presence of following blocking H

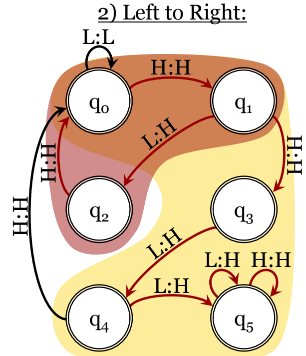
Copperbelt Bemba tone spreading:
- Is a case of **false sour grapes** spreading
- Can be characterized as a weakly deterministic input-output mapping

- Copperbelt Bemba: H spreading to two following tone bearing units provides predictable substring that can be used to mark up final H as successful trigger of unbounded tone spreading
- **Zone of predictability:** predictable substring local to potential trigger of spreading that can be utilized for mark-up

1) Right to Left:



2) Left to Right:



1) Final H (successful trigger) marked up with predictable substring HHLL
2) HHLL maps to HHHH, followed only by Hs

1) Non-final H (unsuccessful trigger) marked up with predictable substring HLH
2) HLH maps to HHH

- **False sour grapes:** zone of predictability local to potential trigger allows transducer to distinctly mark up triggers and non-triggers of unbounded tone spreading

True Sour Grapes Spreading

- **True sour grapes:** no zone of predictability local to potential trigger results in mapping that is not weakly deterministic
- Potential undergoer U preceded at any distance by trigger T assimilates to the trigger

T U U U # ⟶ T T T T #

- If blocker B appears anywhere after a trigger T, no potential undergoers U assimilate to the trigger

T U U B # ⟶ T U U B #

- Successful mark-up strategy must distinguish unsuccessful triggers T_U (T followed by blocker) from successful triggers T_S (T not followed by blocker)

T ⟶ T_U/__(U,T)$_0$B   T ⟶ T_S/__(U,T)$_0$#

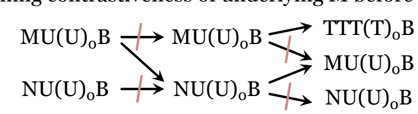
- Using only symbols in alphabet, right to left transducer must mark up successful trigger and surrounding symbols (e.g., T and two following symbols) as some substring M (e.g., TUT)

T(U,T)(U,T) ⟶ M/__(U, T)$_0$#

- Left to right transducer triggers spreading from M and all symbols in M surface as T

U ⟶ T/M(U,T)$_0$__   M ⟶ T T T

- But without **zone of predictability** local to potential trigger, there is no mark-up substring M that we can use while maintaining contrastiveness of underlying M before blockers



True sour grapes spreading cannot be rendered weakly deterministic using a zone of predictability

Conclusion & Future Work

- Main claim: sour-grapes-like patterns of spreading are only attested if they involve **zones of predictability**, rendering their mappings weakly deterministic
- Copperbelt Bemba tone spreading represents a case of weakly deterministic **false sour grapes** spreading
- Possible additional cases of false sour grapes:
  - Tutrugbu ATR harmony (McCollum et al. 2018)
  - Tuyuca nasal harmony? (Barnes 1996)
- Open questions for future work:
  - Do learners (and learning algorithms) make use of zones of predictability?
  - How do zones of predictability affect computational complexity of other phonological processes?

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